The Ideal form of the PID controller is:

$$u(t)=MV(t)=K_p e(t)+K_i \int_o^t e(\tau)\,d\tau+K_d \frac{d}{dt} e(t)$$

Where
Kp: proportional gain, a tuning parameter
Ki: Integral Gain, a tuning parameter
Kd: Derivative gain, a tuning parameter
e: error = Desired out – measured out
t: Time or instantaneous time (the present)
τ: Variable of integration; takes on values from time 0 to present t.

If we take the Z-transform of the Ideal form we get:

$$U(z)=\left[K_p+\frac{K_i}{1-z^{-1}}+K_d\left(1-z^{-1}\right)\right]E(z)$$

If we rearrange it gives us

$$U(z)=\left[\frac{\left(K_p+K_i+K_d\right)+\left(-K_p-2K_d\right)z^{-1}+K_d z^{-2}}{1-z^{-1}}\right]E(z)$$

So if we define

$$A_1=K_p+K_i+K_d A_2=-K_p-2K_d A_3=K_d$$

The equation can be rewritten as

$$U(z)-z^{-1}U(z)=\left[A_1+A_2 z^{-1}+A_3 z^{-2}\right]E(z)$$

This can be converted to:

$$u(t)=u(t-1)+A_1 e(t)+A_2 e(t-1)+A_3 e(t-2)$$

This form is much easier to compute with a microcontroller or DSP. The important thing to remember is the relationship between K_p, K_i, and K_d and A_1, A_2, and $A_{3.}$ All the algorithms and methods for tuning a PID controller assume K_p, K_i, and K_d and not A_1, A_2, and $A_{3.}$

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